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$$D_{m+k,\,m}=\binom{m+k}{k},\quad D_{nm}=\binom{n}{m} \text{ if } n>m.$$

Hence the inverse of transformation T is

$$T^{-1}: x_i=\sum_{j=i}^{g-1}(-1)^{i+j}\binom{j}{i}\xi_j \quad (i=0,\ 1,\ \dots,\ g-1).$$

Since the product of the two transformations is the identity, we have

$$\sum_{j=i}^{l}(-1)^{l+j}\binom{l}{j}\binom{j}{i}=\delta_{il} \quad (\delta_{ii}=1,\ \delta_{il}=0 \text{ if } i\neq l).$$

Conversely, from this well known formula (cf. Netto, p. 255, (43)), follows the evaluation of the determinant D.

289. Proposed by S. A. COREY, Hiteman, Iowa.

Prove that $\left(\frac{n+1}{(n+1)^2-1}+\frac{1}{3}\cdot\frac{n+3}{(n+3)^2-1}+\frac{1}{5}\cdot\frac{n+5}{(n+5)^2-1}+\dots\right)$ $+\left(\frac{2}{3}\cdot\frac{1}{n+2}+\frac{4}{15}\cdot\frac{1}{n+4}+\frac{6}{35}\cdot\frac{1}{n+6}+\dots\right)=\frac{n-1}{(n-1)^2-1}+\frac{1}{3}\cdot\frac{n-3}{(n-3)^2-1}$ $+\frac{1}{5}\cdot\frac{n-5}{(n-5)^2-1}+\dots+\frac{1}{l}\cdot\frac{n-l}{(n-l)^2-1}$, n being any odd integer greater than 1 and $l=n-2$.

Solution by the PROPOSER.

The term by term product of the right hand members of the two Fourier's sine series,

$$1=\frac{4}{\pi}\left[\frac{\sin x}{1}+\frac{\sin 3x}{3}+\frac{\sin 5x}{5}+\dots\dots\right] \text{ and}$$

$$\cos x=\frac{4}{\pi}\left[\tfrac{2}{3}.\sin 2x+\tfrac{4}{15}.\sin 4x+\tfrac{6}{35}.\sin 6x+\dots\dots\right]$$

being equal to the product of the left hand members, may be thus written:

$$\cos x=C_0+C_1\cos x+C_2\cos 2x+C_3\cos 3x+\dots\dots$$

the C's being constants and functions of the coefficients of the sine terms. But as this product of the right hand members is of the same form as the regular Fourier's cosine series for $\cos x$, it must be identical with the latter series, and, hence, each C except C_1 must be equal to zero. Each C with even numbered subscript is identically zero, and in order that the remaining C's may be zero, the equation of the problem must hold.

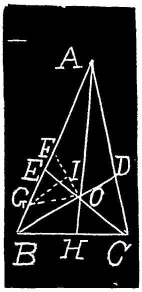
GEOMETRY.

123. Proposed by P. C. CULLEN, Indianola, Iowa.

If the bisectors of two angles of a triangle are equal, those angles are equal, and the triangle is isosceles.

Another Demonstration by JOHN G. GREGG, Terre Haute, Indiana.

Let ABC be the given triangle, and BD, CE, and AH the three bisectors of its angles meeting in O, and let $BD=CE$. We are to show that $\angle ABC=\angle ACB$. If these angles are not equal suppose $\angle ACB>\angle ABC$, then will AB be greater than AC. Take $AG=AC$, and $AF=AD$; then will $OG=OC$ and $OF=OD$, and $\angle GOF=\angle COD=\angle BOE$. Also $OB+OF=BD$...(1), and $OG+OE=CE$...(2).



It can be established that E will always fall between G and F. OB is greater than OG, and OF is greater than, equal to, or less than OG. If $OF>OG$, then also $OF>OE$, and $OB+OF>OG+OE$, or by (1) and (2), $BD>CE$. But by hypothesis, $BD=CE$. Hence $\angle ACB=\angle ABC$. Q. E. D.

Again, if OF is equal to or less than OG, draw GI making $\angle OGI=\angle OBA$. Obviously I will fall between O and F, and the triangles OBE and OGI are similar. Then since $BO>OG$, we have $BO-OE>OG-OI$, and much more, $BO-OE>OG-OF$. Hence $BO+OF>OG+OE$, and from (1) and (2), $BD>CE$ as before, and the theorem is established.

COROLLARY 1. If two lines BD and CE are drawn through a point O in the bisector of an angle, and meeting the sides of the angle, the one (BD) making the less angle with the bisector is the greater.

COROLLARY 2. If the two lines BD and CE make equal angles with the bisector, they are equal.

COROLLARY 3. The line through O, perpendicular to the bisector, is a minimum.

COROLLARY 4. Two triangles are equal if their bases, the angles opposite the bases, and the bisectors of those angles are respectively equal.

320. Proposed by J. SCHEFFER, A. M., Kee Mar College, Hagerstown, Md.

Prove by plane geometry the following interesting theorem:

If from a point in the plane of a triangle perpendiculars are demitted upon the three sides of the triangle, and if the area of the triangle formed by connecting the feet of these perpendiculars is denoted by $\triangle'$, *the distance of the assumed point from the center of the circle circumscribed about the original triangle by* R', *the radius of the circumscribed circle by* R, *and the area of the pedal triangle by* $\triangle$, *then will* $\triangle'/\triangle=\pm[(R^2-R'^2)/R^2]$.